Sanjay Ghodawat University, Kolhapur
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2020-21

EXM/P/09/00

Year and Program: **F.Y. M. Sc.** | **School of Science** | **Department: Mathematics**

Course Code: MTS513 | **Course Title: Differential Geometry** | **Semester : I**

Day : **Date:** | **End Semester Examination** | **Time:2 Hrs.** **Max Marks:50**

Instructions: 1) All Questions are compulsory.
2) Use of non-programmable calculator is allowed

		Marks	Bloom's Level	CO
Q.1	**Answer the following (any Two)**			
a)	Define directional derivative in the direction of tangent vector. If f and g are real valued function defined on $\mathbb{R}^3$ and v_p and w_p are tangent vectors to $\mathbb{R}^3$ at point p, then show that i) $(aV_p + bW_p)[f] = aV_p[f] + bW_p[f]$ ii) $V_p[af + bg] = aV_p[f] + bV_p[f]$ ii) $V_p[fg] = V_p[f].g(p) + f(p).V_p[g]$ where a and b are real numbers	8	L1	CO1
b)	Show that, the curves defined by $\alpha = (t, 1+t^2, t), \beta = (\sin t, \cos t, t)$ and $\gamma = (\sinh t, \cosh t, t)$ have the same initial velocity $\overline{V_p}$. If $f = x^2 - y^2 + z^2$, compute $\overline{V_p}[f]$ by evaluating f and each of the curves.	8	L3	CO1
c)	Let β be a unit speed curve in $\mathbb{R}^3$ with curvature $\kappa > 0$. Show that the vector fields $\overline{T}, \overline{N}, \overline{B}$ at every point β forms a Frenet frame fields on β.	8	L2	CO2
Q.2	**Answer the following (any two)**			
a)	If $\phi = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$ is a 1-form then find $d\phi$ and hence show that $d^2\phi = 0$.	08	L3	CO2
b)	Determine the curvature of the ellipse given by $\alpha(t) = (a\cos t, b\sin t, 0)$ and hence deduce the curvature of a circle of radius r from it. Also find the torsion τ, $\overline{T}, \overline{B}$ and $\overline{N}$.	08	L2	CO3
c)	Show that every isometry of $\mathbb{R}^3$ can be uniquely described as an orthogonal transformation followed by translation.	08	L3	CO3

Q.3 Answer the following (any two)

a)	Define Co-ordinate patch and proper co-ordinate patch. Let $X : D \to \mathbb{R}^3$, be defined by $X(u,v) = (u^2, uv, v^2), \forall (u,v) \in D$ where $D = \{(u,v) \in \mathbb{R}^2 / u > 0, v > 0\}$. Show that X is a co-ordinate patch. Is it proper? Justify your claim.	09	L2	CO4
b)	i) Show that a surface obtained by rotating a curve is a surface of revaluation.	05	L5	CO4
	ii)Prove that a geodesic curve has constant speed.	04	L3	CO5
c)	If $\bar{v}$ and $\bar{w}$ are linearly independent tangent vectors at point p of M then show that i) $S(\bar{v}) \times S(\bar{w}) = K(p)(\bar{v} \times \bar{w})$ ii) $S(\bar{v}) \times \bar{w} + \bar{v} \times S(\bar{w}) = 2H(p)(\bar{v} \times \bar{w})$ And hence derive formulae of $K(p)$ and $H(p)$.	09	L4	CO5

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